Magnetometer Bias Determination and Spin-Axis Attitude Estimation for the AMPTE Mission

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Introduction

THIS Note describes methods for determining spin-axis attitude (i.e., the direction in space of the spacecraft spin axis) and magnetometer biases which have been investigated for ground support of the Active Magnetospheric Particle Tracer Explorer (AMPTE) mission.

The AMPTE mission will consist of two spacecraft.[1] The first is the Ion Release Module (IRM), provided by the Federal Republic of Germany, which will be placed in a highly elliptical orbit with apogee at approximately 19 Earth radii in order to release lithium tracer ions outside the magnetosphere. This spacecraft will be spin stabilized at a rate of 30 rpm. The second spacecraft is the Charge Composition Explorer (CEE), which will detect the tracer ions inside the magnetosphere at altitudes of from 300 km to 7.5 Earth radii. The CCE will be spin stabilized at 10 rpm.

Estimation of spin-axis attitude for both AMPTE spacecraft will be based on the measurements of the geomagnetic field and the projection of the Sun line on the spacecraft spin-axis, which we take nominally to be the symmetry axis $\boldsymbol{Y}_A$ of the spacecraft bus.

For the purpose of this study, the attitude sensors are assumed to consist of a three-axis magnetometer and a Sun sensor which measures the angle between the Sun line and $\boldsymbol{Y}_A$. For simplicity it is assumed likewise that one axis of the magnetometer is along $\boldsymbol{Y}_A$. The other two axes of the magnetometer define $\hat{\boldsymbol{X}}_A$ and $\hat{\boldsymbol{Z}}_A$.

The measured quantities are taken to be

$\boldsymbol{M}$ = magnetic field vector in body coordinates

$\cos\beta = \hat{\boldsymbol{S}}\cdot\hat{\boldsymbol{Y}}_A$, where $\hat{\boldsymbol{S}}$ is the unit vector directed from the spacecraft to the Sun (β is the "Sun angle").

Attitude determination activities fall into two areas: determination of spin-axis atttitude and determination of the magnetometer biases.

Because the apogee for these two spaceraft is so great, accurate geomagnetic field data for attitude estimation are available only for the segment of the orbit near perigee. This is due to the poor accuracy of the magnetic-field model at such high altitudes, which results from both the small magnitude of the geomagnetic field as well as fluctuations in the field caused by extraterrestrial phenomena. However, because of the large spacecraft angular momenta, it can be assumed for both spacecraft that the spin-axis attitude at apogee will not differ markedly from that at perigee of the same orbit.

Algorithms for spin-axis attitude and magnetometer bias determination are now being investigated. These are 1) estimation of three-axis magnetometer bias and 2) estimation of spin-axis attitude from measurements of the Sun and geomagnetic field angle. Each of these algorithms is a batch estimator utilizing a long segment of magnetometer and Sun data. The algorithms are developed in succeeding sections and then tested using simulated AMPTE data.

Magnetometer Bias Determination

Generally, the magnetometer biases must be determined very soon after injection of the spacecraft into its orbit and before attitude information becomes available. Therefore, a bias determination procedure must be developed which is independent of the attitude. The quantities available for the estimation procedure are:

$\boldsymbol{H}_i$ = the model magnetic field in the geocentric inertial (GCI) cordinate system at time i

$\boldsymbol{M}_i$ = magnetometer reading at time i

$\boldsymbol{B}$ = the magnetometer bias vector

For the ith point, the field-magnitude error $\delta_i(\boldsymbol{B})$ is defined by

$$\delta_i(\boldsymbol{B}) = |\boldsymbol{H}_i|^2 - |\boldsymbol{M}_i - \boldsymbol{B}|^2 \tag{1}$$

In the absence of measurement and modeling errors a value of the magnetometer bias vector can be found for which all field-magnitude errors vanish. Otherwise, the optimal value of $\boldsymbol{B}$ is that which minimizes the loss function

$$L(\boldsymbol{B}) = \frac{1}{2}\sum_{i=1}^{N}\omega_i\,|\delta_i(\boldsymbol{B})|^2 \tag{2}$$

where ω_i is the weight associated with the ith data point. The weights are assumed to be normalized to have unit sum

$$\sum_{i=1}^{N}\omega_i = 1 \tag{3}$$

At the optimal estimate, $\hat{\boldsymbol{B}}$, of the magnetometer bias vector

$$\left.\frac{\partial L}{\partial \boldsymbol{B}}\right|_{\hat{\boldsymbol{B}}} = -2\sum_{i=1}^{N}\omega_i\delta_i(\hat{\boldsymbol{B}})(\hat{\boldsymbol{B}}-\boldsymbol{M}_i) = \boldsymbol{0} \tag{4}$$

Equation (4) may be recast to read

$$G\hat{\boldsymbol{B}} = \boldsymbol{b} + \boldsymbol{F}(\hat{\boldsymbol{B}}) \tag{5}$$

where

$$G = (\langle|\boldsymbol{H}|^2\rangle - \langle|\boldsymbol{M}|^2 \geq) I - 2\langle \boldsymbol{M}\boldsymbol{M}^T\rangle \tag{6a}$$

$$\boldsymbol{b} = \langle(|\boldsymbol{H}|^2 - |\boldsymbol{M}|^2)\boldsymbol{M}\rangle \tag{6b}$$

$$\boldsymbol{F}(\boldsymbol{B}) = |\boldsymbol{B}|^2\langle \boldsymbol{B}-\boldsymbol{M}\rangle - 2\boldsymbol{B}\cdot\langle \boldsymbol{M}\rangle\boldsymbol{B} \tag{6c}$$

The bracket denotes the weighted average

$$\langle A\rangle = \sum_{i=1}^{N}\omega_i A_i \tag{7}$$

The superscript T denotes the matrix transpose, and the symbol I denotes the 3×3 identity matrix.

Equation (5) can be solved iteratively to obtain the best value for the bias vector according to

$$\hat{\boldsymbol{B}}_0 = \boldsymbol{0} \tag{8a}$$

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$$\hat{B}_{k+1} = G^{-1}[b + F(\hat{B}_k)] \tag{8b}$$

where $\hat{B}_k$ denotes the kth estimate of B. The iteration is terminated when

$$|\hat{B}_k - \hat{B}_{k-1}| / |\hat{B}_{k-1}| < \epsilon \tag{9}$$

where ϵ is some arbitrarily small value determined by the accuracy requirements of the mission.

For $|B| \ll |H|$, this fixed-point iterative method converges. As a rule, the convergence is slowest for the component of B along the spin axis of the spacecraft (because G generally has its smallest eigenvalue along that direction). For $|B| \gg |H|$, the algorithm will, in general, not converge.

In simulations[2,3] the algorithm was found to converge more slowly than the method currently in use in support of NASA near-Earth missions,[4] which solves Eq. (4) iteratively by the Newton-Raphson method. However, while convergence is slower with the present algorithm, it occurred in all trials while the Newton-Raphson method was not always able to converge to a solution for the case of the highly eccentric AMPTE orbit, where usable magnetometer data are available for only a small fraction of the orbit near perigee.

Spin-Axis Attitude Determination

Once the magnetometer biases have been chosen properly, data from the Sun sensor and the magnetometers may be used to determine the spin-axis attitude. It is assumed that the spin axis is constant over the data interval examined.

The spin axis is denoted by a. The data are

β_i = measured Sun angle at time i $\quad i = 1,\ldots,N_S$

M_i = measured magnetic field vector at time i, (corrected for any magnetometer bias) $\quad i = 1,\ldots,N_M$

$\hat{S}_i$ = (true) Sun vector in GCI at time i, measured from the spacecraft to the Sun $\quad i = 1,\ldots,N_S$

H_i = (true) geomagnetic field in GCI at time i, $\quad i = 1,\ldots,N_M$

There is no requirement of simultaneous Sun-sensor and magnetometer data.

The spin-axis (attitude) vector, a, is subject to the constraint

$$a \cdot a = 1 \tag{10}$$

and, therefore, the spin-axis vector is chosen to minimize the loss function

$$L(a) = \frac{1}{2} \sum_{i=1}^{N_S} \omega_S(i)\, |a \cdot \hat{S}_i - \cos\beta_i|^2 + \frac{1}{2} \sum_{i=1}^{N_M} \omega_M(i)\, |a \cdot \hat{H}_i - \cos\eta_i|^2 - \frac{1}{2}\lambda a \cdot a \tag{11}$$

where

λ = Lagrange multiplier chosen to satisfy the constraint equation

$\omega_S(i)$ = weight assigned to the ith Sun vector measurement

$\omega_M(i)$ = weight assigned to the ith magnetic field measurement

The quantity η is the angle between the geomagnetic field and the spacecraft spin axis given by

$$\eta = \cos^{-1}(M_y / |M|) \tag{12}$$

and the weights are normalized to have unit sum

$$\sum_{i=1}^{N_S} \omega_S(i) + \sum_{i=1}^{N_M} \omega_M(i) = 1 \tag{13}$$

The optimal estimate of the spin-axis vector, $\hat{a}$, is a solution of

$$\left.\frac{\partial L}{\partial a}\right|_{\hat{a}} = \sum_{i=1}^{N_S} \omega_S(i)\, (\hat{a} \cdot \hat{S}_i - \cos\beta_i)\hat{S}_i + \sum_{i=1}^{N_M} \omega_M(i)\, (\hat{a} \cdot \hat{H}_i - \cos\eta_i)\hat{H}_i - \lambda\hat{a} = 0 \tag{14}$$

The solution to Eq. (14) may be written as

$$\hat{a} = (A - \lambda I)^{-1} c \tag{15}$$

where

$$A = \langle \hat{S}\hat{S}^T \rangle_S + \langle \hat{H}\hat{H}^T \rangle_M \tag{16a}$$

$$c = \langle \cos\beta \hat{S} \rangle_S + \langle \cos\eta \hat{H} \rangle_M \tag{16b}$$

and the brackets denote weighted averages over the magnetometer or Sun data. That is,

$$\langle C \rangle_K \equiv \sum_{i=1}^{N_K} \omega_K(i) C_i \tag{17}$$

where K denotes either M or S.

The value of the Lagrange multiplier will be given by the root of

$$f(\lambda) \equiv c^T (A - \lambda I)^{-2} c - 1 \tag{18}$$

for which the loss function is smallest. As a rule, this will be the root of Eq. (18) which is smallest in magnitude.

The solution for $\hat{a}$ may be computed iteratively as

$$\hat{a}_k = (A - \lambda_k I)^{-1} c \tag{19}$$

where the sequence λ_k is given by

$$\lambda_0 = 0 \tag{20a}$$

$$\lambda_{k+1} = \lambda_k + (1 - \hat{a}_k \cdot \hat{a}_k)/2\hat{a}_k^T (A - \lambda_k I)^{-1} \hat{a}_k \tag{20b}$$

and the iteration is terminated when $|\hat{a}_k - \hat{a}_{k-1}|$ becomes less than some preassigned value. The estimate $\hat{a}_k$ will be a unit vector only in the limit $k \to \infty$ and, therefore, should be renormalized when the sequence is truncated. In general, λ is expected to be quite small since a is already overdetermined without the normalization constraint. If σ is the typical standard deviation of β or η, then it may be expected naively that

$$|\lambda| \approx \sigma^2 / (N_S + N_M) \tag{21}$$

The method currently in use[5] parameterizes the spin-axis vector in terms of spherical angles (θ, ϕ) and minimizes the loss function of Eq. (11) without the constraint term. Convergence is fast once the trial estimate is within a small neighborhood of the solution. However, the method currently in use does not provide a good starting value. Such a value can be inferred from the zero-th order estimate given by Eq. (19) above[6] (appropriately normalized) through that starting value is not apparent without recourse to the above derivation.

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Modification of Pitching Stability by an Atmospheric Wave

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Introduction

ATMOSPHERIC disturbances can modify the flight characteristics of aircraft. Extreme examples of this are the destructive consequences of downbursts[1] and the disruptive effects of CAT (clear air turbulence). Wind shear and turbulence are particularly unwelcome in the phases of take-off and landing, where they may critically interfere with aircraft control.[2,3,4,5] In this Note a related effect is demonstrated—the modification of the high-frequency oscillation when an aircraft encounters a coherent wave disturbance. The effect is most noticable when the wave frequency, seen from the aircraft, is twice the frequency of the free pitching mode. The interaction modifies the damping rate of the motion and leads to excitation of all harmonics of the pitching frequency. The effect is something of a curiosity, in that while it may enhance aircraft vibration a little, it should not lead to instability unless the aircraft is already dangerously underdamped.

The atmospheric waves are internal gravity waves of the gravity shear or Kelvin-Helmholtz variety.[6] It is relatively common for aircraft to encounter such waves, since flight paths often coincide with the jet streams, favored locations for shear waves. However, the more extreme instances of wave activity are likely to be avoided, since they coincide with servere CAT.[7]

Model Calculation

The calculation is an elementary study of the longitudinal pitching mode of an aircraft flying with fixed controls. Severe approximations to the dynamics and total absence of any control corrections make this calculation "preliminary" at best. Its advantage is simplicity, in both presentation and interpretation.

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In its simplest form, the "high frequency" mode is defined by the aircraft behavior in a uniform atmosphere (e.g. Seckel[8]). It has also been studied in the presence of a horizontally homogeneous vertical shear.[5] The present study allows periodic variations in the wind field. Aircraft response has been obtained in several models with various degrees of complexity. The most complex has been a two-air-foil rigid craft in free longitudinal motion with coupled plunging and pitching. But the essential result is easily illustrated by isolating the perturbation moment generated by the lift of the tail air foil. This most simple model produces essentially the same pitching behavior as the most complicated formulation. Perhaps this is not surprising in that it reflects the dominant role of lift at the tail air foil in all matters of pitching stability. The simplest formulation is presented in this Note.

Formulation

Figure 1 defines the geometry of the system. The aircraft is trimmed for level flight at constant air speed S when it encounters a periodic wind perturbation. Changes in the magnitude and direction of the tail lift, L, induce pitching through angle θ about the center of gravity of the aircraft:

$$I\ddot{\theta} = \mathrm{d}L\cos(\phi-\theta) - M_0 \tag{1}$$

I is the moment of inertia of the craft and d the effective moment arm of the tail, about the center of gravity. The moment of the tail lift in trimmed level flight, M_0, is balanced by other torques whose perturbations are normally small compared with the right hand side of Eq. (1).

In this model, the center of gravity remains in uniform horizontal motion with constant ground speed. Air speed relative to the tail, V, changes because of perturbations (u, w) in the wind, and the tail rotation velocity $(\mathrm{d}\dot{\theta}\sin\theta,\ \mathrm{d}\dot{\theta}\cos\theta)$. An analogous separation in the dynamics was used by Biggers[9] in his study of the flapping stability of helicopter rotors. After assuming a constant forwards speed for the axis of the rotor system, Biggers obtained an equation with periodic coefficients for the motion of the rotor blades. In this sense also his study resembles the calculation presented below.

Equation (1) is simplified by assuming that the lifting coefficient, $c(\alpha)$, varies linearly with changes in angle of attack α about a mean setting α_0. The dynamics can then be set up with the relations

$$L = \tfrac{1}{2}\rho A\{c_0 + c_l(\alpha-\alpha_0)\}V^2 \tag{2}$$

$$\alpha - \alpha_0 = \phi - \theta \tag{3}$$

$$M_0 = \tfrac{1}{2}\mathrm{d}\rho A c_0 S^2 \tag{4}$$

$$\phi = \tan^{-1}\{(w - \mathrm{d}\dot{\theta}\cos\theta)/(S + \mathrm{d}\dot{\theta}\sin\theta - u)\} \tag{5}$$

$$V^2 = (S + \mathrm{d}\dot{\theta}\sin\theta - u)^2 + (\mathrm{d}\dot{\theta}\cos\theta - w)^2 \tag{6}$$

Combining Eqs. (1)-(4)

$$I\ddot{\theta} = \tfrac{1}{2}\mathrm{d}A\{c_0(V^2\cos(\phi-\theta) - S^2) + c_l(\phi-\theta)V^2\} \tag{7}$$

Now two, distinct, small amplitude approximations are applied. First, all equations are linearized in θ, the disturbance to the pitch. This is the standard approximation of "infinitessimal" stability theory. Second, the wave amplitude only appears as corrections of order u/S and w/S in expression that are of order 1. Since these wave terms are relatively small, all equations are further linearized in these amplitudes. It must be noted that in this procedure response terms of order $\theta\ (u/S)$ are retained while formally larger forcing terms of order $(u/S)\cdot(u/S)$ are discarded. This approach cannot be rigorously justified. But to the extent that